Katayama, S. and Terasaka, H.
Osaka J. Math.
4 (1967), 111-132

ON CONGRUENT AXIOMS IN LINEARLY ORDERED SPACES, II[1)]

SHIGERU KATAYAMA AND HIDETAKA TERASAKA

(Received March 31, 1967)

6. Model M(R, C, I)

M(R, C, I): *A model of a geometry in which Axioms* R, C *and* I *alone hold besides Axiom* E. (Notice that I follows automatically from E, R and C.)

The construction of M (R, C, I) is quite different from those of other models, and its exposition here may be too long, but it seems to the authors appropriate to provide it with a full proof. It depends essentially upon Lemma below, and we will begin by introducing some definitions and auxiliarly axioms needed in it.

Let $\boldsymbol{A}$ be a finite number of linearly ordered points, in which congruence relations are supposed to hold among some of the segments, and let P, Q, P' etc. denote points of $\boldsymbol{A}$.

DEFINITION. We write

$$PQ \approx Q'P' \quad \text{or} \quad Q'P' \approx PQ\,,$$

if and only if

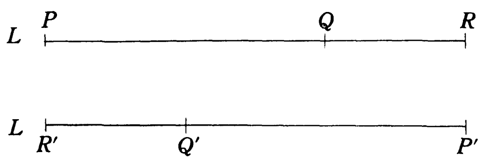
$$PQ = Q'P' \quad \text{and} \quad Q'P' = PQ$$

at the same time.

Axiom E_u: *If* $PQ=Q'P'$ *and* $PQ=Q'P''$, *then* $P'=P''$.

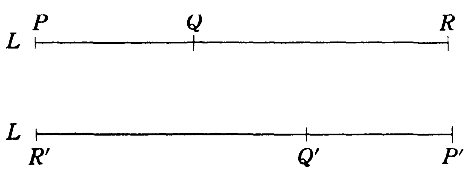
Axiom C^+ (=**Axiom** C)

$$\left.\begin{array}{l} P<Q<R\,, \\ R'<Q'<P'\,, \\ PQ=Q'P'\,, \\ QR=R'Q' \end{array}\right\} \Rightarrow PR=R'P'.$$



Axiom $\tilde{\mathrm{C}}^+$

$$\left.\begin{array}{l} P<Q<R\,, \\ R'<Q'<P'\,, \\ PQ\approx Q'P'\,, \\ QR=R'Q' \end{array}\right\} \Rightarrow \begin{cases} PR\approx R'P' \\ QR\approx R'Q'. \end{cases}$$



1) Continuation of Part I, this Journal, vol. 3 (1966), 269–292. Referred to as Part I.