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ON DIFFERENTIABILITY AND ANALYTICITY OF SOLUTIONS OF WEIGHTED ELLIPTIC BOUNDARY VALUE PROBLEMS

HIROKI TANABE

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The present paper is concerned with the differentiability and analyticity of solutions of weighted elliptic boundary value problems (see [2] for the definition of weighted ellipticity)

$$A(x, t, D_x, D_t)u(x, t) = f(x, t), \qquad x\in\Omega, \tag{0.1}$$

$$B_j(x, t, D_x, D_t)u(x, t) = 0, \qquad x\in\partial\Omega, \ j=1, \cdots, m, \tag{0.2}$$

in some cylindrical domain with Ω as its base, where we denote the order type of A by $(2m, l)$. We first investigate such regularity properties of the solution u considered as a function of t with values in $L^2(\Omega)$ or $H_{2m}(\Omega)$ and then the same properties of u as a numerical function of all independent variables (x, t). In [2] S. Agmon and L. Nirenberg proved the differentiability and analyticity in t of the solutions of (0.1)–(0.2) in $L^p(\Omega)$, $1<p<\infty$, under the corresponding hypothesis on f in case in which all the coefficients of A and $\{B_j\}_{j=1}^m$ do not depend on t with the aid of their general results on abstract differential equations

$$\frac{1}{i}\frac{du}{dt} - Au = f(t) \tag{0.3}$$

in a Banach space. Recently in [4] A. Friedman obtained such kind of regularity theorems for the solutions of abstract differential equations

$$\frac{1}{i}\frac{du}{dt} - A(t)u = f(t) \tag{0.4}$$

in a Hilbert space using Fourier transform in t. In his results $A(t)$ may depend on t but is assumed to have a constant domain. In [11] the author showed that A. Friedman's method can be applied to the problem with time-dependent boundary conditions

$$\partial u(x, t)/\partial t + A(x, t, \partial/\partial x)u(x, t) = f(x, t), \qquad x\in\Omega, \tag{0.5}$$

$$B_j(x, t, \partial/\partial x)u(x, t) = 0, \qquad x\in\partial\Omega, \ j=1, \cdots, m, \tag{0.6}$$