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ON R-ALGEBRAS WHICH ARE R FINITELY GENERATED

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Let K be a field and R a ring with 1. We know several conditions under which an R-algebra is a finitely generated R-module. In [6] Rosenberg and Zelinsky obtained, for a K-algebra A, those conditions in a case where $A\underset{K}{\otimes}A^*/N(A\underset{K}{\otimes}A^*)$ is Artinian, where A^* is an anti-isomorphic algebra of A and $N(*)$ is the radical of $*$.

In §1 we shall study a similar problem in a case where $A\underset{K}{\otimes}A^*$ is Noetherian and obtain, for an algebraic algebra A over K such that $A/N(A)$ is a semi-simple ring with minimum condition, that $[A:K]<\infty$ if and only if $A\underset{K}{\otimes}A^*$ is right Noetherian.

In §2 we consider a primitive K-algebra with minimal one sided ideals. We give a condition that the associated division ring is of a finite K-dimension.

Finally we consider a separable R-algebra A which is a submodule in a free R-module. If R is Noetherian, then we show that A is R-finitely generated as R-module.

1. Algebras of finite type

In this paper we always assume that K means a field and R a commutative ring with 1.

Let $A_2\supseteq A_1$ be R-algebras. Then we have a natural homomorphism Φ: $A_1\underset{R}{\otimes}A_1^*\rightarrow A_2\underset{R}{\otimes}A_2^*$. We denote also the image of Φ by $A_1\underset{R}{\otimes}A_1^*$ if there are no confusions. Furthermore, we have a natural right $A_i\underset{R}{\otimes}A_i^*$-homomorphism φ_i: $A_i\underset{R}{\otimes}A_i^*\rightarrow A_i$ by setting $(a\otimes b^*)=ba$. We denote its kernel by J_i.

The following lemma is based on a suggestion of M. Auslander.

Lemma 1. *Let A_3 be an R-algebra and $A_2\supsetneqq A_1$ proper R-subalgebras contained in the center of A_3. We assume that A_{i+1} is A_i-projective for $i=1,2$. Then $J_3\supsetneqq J_2A_3^e\supsetneqq J_1A_3^e$, where $A_3^e=A_3\underset{R}{\otimes}A_3^*$.*