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ON THE THEORY OF VARIATION OF STRUCTURES DEFINED BY TRANSITIVE, CONTINUOUS PSEUDOGROUPS

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The theory of deformations of structure was begun some years ago by Kodaira and Spencer [6], who laid the foundations for the theory of variation of complex structure. Later, together with Nirenberg, they established the fundamental existence theorem. On the other hand, it was soon realized that the deformations of complex structure was (at least conceptually) a special case of the theory of variation of the structure defined by a transitive, continuous pseudo-group Γ acting on a manifold X (briefly, we say a Γ-structure on X). In the complex analytic case, $\Gamma=\Gamma_{GL(n,C)}$ is the pseudo-group of all local bi-holomorphic transformations in $\boldsymbol{C}^n$. The theory of deformations of other special Γ-structures has been discussed in [4] and [6]. Following this, Spencer succeeded in [7] in establishing the basic mechanism for deformations of what might be called flat Γ-structures; i.e. Γ-structures for which a certain jet bundle has coordinate cross-sections of a suitable type. Furthermore, he proved the existence theorems in case $\Gamma\subset\Gamma_{GL(n,C)}$. On the other hand, in [1] (see also [9]), the deformation theory of certain non-flat Γ-structures has found important applications, and, in [3], the general theory of a (perhaps non-transitive) Γ-structure has been developed. The existence theorems for certain special cases have also been given in [3].

The purpose of the present paper is to discuss the deformations of a general Γ-structure. That this is possible is not surprising, and it may even be possible to extend [7] to the general case. However, we have chosen a somewhat different approach based on the following heurestic consideration: Let X have a Γ-structure and suppose that this structure is given by a certain "Maurer-Cartain form" Φ defined on a suitable principal bundle $Q\to X$. Thus, Φ is a Lie algebra valued one form on Q which satisfies the Maurer-Cartan equation.

(1) $d\Phi-\frac{1}{2}[\Phi, \Phi]=0\,.$