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ON THE GROUPS WITH THE SAME TABLE OF CHARACTERS AS ALTERNATING GROUPS

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1. Introduction

It was proved by H. Nagao that a finite group which has the same table of characters as a symmetric group S_n is isomorphic to S_n. The purpose of this paper is to prove the following theorem.

Theorem. *If a finite group G has the same table of characters as an alternating group A_n, then G is isomorphic to A_n.*

As is shown in [2], a group G as in the theorem has the same order as A_n, therefore the theorem is trivial for $n=2$ and 3. Furthermore, the degrees of corresponding irreducible characters of G and A_n coincide with each other, the numbers of elements of corresponding conjugate classes of G and A_n are the same, and G has the same multiplication table of conjugate classes as A_n. From the last fact it follows that G is simple for $n\geqq 5$. Since it is known that a simple group of order 60 or 360 is isomorphic to A_5 or A_6, the theorem is true for $n=5$ and 6.

Now we shall give here an outline of the proof of the theorem which will be given in the next section. An alternating group A_n is isomorphic to the group generated by $a_1, a_2, \cdots, a_{n-2}$ with the following defining relations;

$$(*)\begin{cases} a_1^3=1,\ a_2^2=a_3^2=\cdots=a_{n-2}^2=1 \\ (a_ia_{i+1})^3=1 \qquad (i=1,2,\cdots,n-3) \\ (a_ia_j)^2=1 \qquad (i=1,2,\cdots,n-4,\ i+1<j) \end{cases}$$

(For the proof, see [1], Note C). The proof of the theorem is carried out by showing the existence of elements $a_1,\cdots,a_{n-2}$ in G which satisfy the above relations.

Let $C^*(i_1^{\alpha_1}, i_2^{\alpha_2},\cdots)$ be the totality of elements of A_n which can be expressed as a product of α_1 cycles of length i_1, α_2 cycles of length $i_2,\cdots$ such as each of letters occurs in only one cycle of them, where we as-