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FACTORIZATION OF DIFFERENTIAL OPERATORS AND DECOMPOSITION OF SOLUTIONS OF HOMOGENEOUS EQUATIONS

To Professor Y. Akizuki on his 60-th birthday

By

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§ 1. Introduction

Let Ω be an open set in ν-dimensional Euclidean space $\boldsymbol{R}^\nu$ whose points are described by a fixed coordinate system $x=(x_1, \cdots, x_\nu)$. Let $L(X)$ be a polynomial of ν-variables $X=(X_1, \cdots, X_\nu)$ with complex coefficients. Replacing X by partial differentiations $D=(D_1, \cdots, D_\nu)$, $D_k=\frac{1}{i}\frac{\partial}{\partial x_k}$ $(i=\sqrt{-1})$, we get a partial differential operator with constant coefficients $L(D)$. Let $\mathcal{D}'(\Omega)$ be the space of distributions defined in Ω (See L. Schwartz [11]) and take a linear subspace E of $\mathcal{D}'(\Omega)$ which is stable under the operations of partial differentiations.

Let us consider a differential equation of the form

$$L(D)u = 0 \tag{1.1}$$

where u is an unknown element of E.

When we have a factorization of $L(X)$ into *mutually prime* factors[1)]

$$L(X) = P(X)Q(X)\,, \tag{1.2}$$

it is very common in applied mathematics to seek for a general solution of (1.1) in the form of a sum

$$u = u_1+u_2, \qquad u_1, u_2 \in E\,, \tag{1.3}$$

where u_1 and u_2 are solutions of the equations corresponding to the factors, i.e.

$$\begin{aligned} P(D)u_1 &= 0\,, \\ Q(D)u_2 &= 0, \text{ in } \Omega\,. \end{aligned} \tag{1.4}$$

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1) Factorizations are always considered in the polynomial ring $\boldsymbol{C}[X_1, \cdots, X_\nu]$ over the complex number field $\boldsymbol{C}$.