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LOCAL TRIANGULATION OF REAL ANALYTIC VARIETIES

Dedicated to Professor K. Shoda on his sixtieth birthbay

By

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Introduction

In the recent study of real analytic varieties, one of the main problems is to decompose a given variety into some reasonable subsets. H. Whitney proved that a real algebraic variety can be expressed as a union of mutually disjoint manifolds of various dimensions [6], while A. H. Wallace decomposed it into sheets (analytically connected sets) [4]. Later, Whitney and F. Bruhat extended Whitney's result to the case of so-called C-analytic varieties [7], and Wallace also generalized his result to real analytic varieties in somewhat milder form [5]. In these studies, local connectivity of real analytic varieties (see, for example, [7], Prop. 2) plays a fundamental role.

In our present paper, we first prove that a real analytic variety E is locally triangulable with given subvarieties as subcomplexes (Theorem 1), from which local connectivity follows immediately, and as a consequence of this Theorem, we show that a real algebraic variety is globally triangulable into a finite number of simplexes (Theorem 2). Next, in a global vein, we show that a real analytic variety admits, what we call, pseudo-cell docomposition (Corollary to Theorem 3).

When we carry out the proof by induction, the main difficulty lies in the fact that a (local) projection of E on a subspace (with respect to a coordinate system) is not necessarily a variety, even though a coordinate system is p-proper (see § 1) for the (local) complexification E^* of E. The most part of our proof is devoted to eliminate this difficulty. The idea of our proof is to get a (local) triangulation of E as a subcomplex of a bigger complex $\tilde{G}$ which has a more convenient form than E itself. To do so, we first construct two imbedding varieties $\tilde{E}^*$ and $\hat{E}^*$ which locally contain E in such a way that $\hat{E}^*$ contains the real part of $\tilde{E}^*$ (Lemma 1). Next we introduce the notions of p-proper simplex and p-proper complex (§ 4 and § 5) and show that the triangulation of p-proper complex can be extended to the whole neighborhood (Lemma 3). Taking the real