Kumano-go, H.
Osaka Math. J.
14 (1962), 181–212.

ON THE UNIQUENESS OF THE SOLUTION OF THE CAUCHY PROBLEM AND THE UNIQUE CONTINUATION THEOREM FOR ELLIPTIC EQUATION

By

Hitoshi KUMANO-GO

§ 0. Introduction. We shall consider differential operators with complex valued coefficients in a neighborhood of the origin in the $(\nu+1)$-dimensional Euclidean space whose points are denoted by $(t, x)=(t, x_1, \cdots, x_\nu)$ or $(r, \theta)=(r, \theta_1, \cdots, \theta_\nu)$ or simply $(x)=(x_1, \cdots, x_{\nu+1})$.

The object of this note is to prove the following two theorems by a unified method.

The one is the theorem on the uniqueness of the solution of the Cauchy problem for the differential equation of the form

$$Lu \equiv \sum_{i+|\mu|\leqq m} a_{i,\mu}(t, x)\frac{\partial^{i+|\mu|}}{\partial t^i \partial x^\mu}u(t, x) = f(t, x) \tag{0.1}$$

$(\mu=(\mu_1, \cdots, \mu_\nu), |\mu|=\mu_1+\cdots+\mu_\nu; x=(x_1, \cdots, x_\nu), \partial x^\mu=\partial x_1^{\mu_1}\cdots\partial x_\nu^{\mu_\nu})$ under the following conditions: Set $L_m \equiv \sum_{i+|\mu|=m} a_{i,\mu}(t, x)\frac{\partial^m}{\partial t^i \partial x^\mu}$. We assume that the associated characteristic polynomial $L_m(t, x, \lambda, \xi)=\sum_{i+|\mu|=m} a_{i,\mu}(t, x)\lambda^i\xi^\mu$ $(\xi=(\xi_1, \cdots, \xi_\nu), \xi^\mu=\xi_1^{\mu_1}\cdots\xi_\nu^{\mu_\nu})$ can be written as

$$L_m(t, x, \lambda, \xi') = \prod_{i=1}^{k}(\lambda-\lambda_i^{(1)}(t, x, \xi'))\prod_{j=1}^{m-k}(\lambda-\lambda_j^{(2)}(t, x, \xi')) \quad (0\leqq k\leqq m) \tag{0.2}$$

for ξ' in some neighborhood of any ξ_0' on the unit sphere $S=\{\xi'; |\xi'|=1\}$ $(|\xi'|=(\sum_{i=1}^{\nu}\xi_i'^2)^{1/2})$ and for (t, x) in some neighborhood of the origin where $\lambda_i^{(1)}=-q_i^{(1)}+ip_i^{(1)}$ $(i=1, \cdots, k)$ and $\lambda_j^{(2)}=-q_j^{(2)}+ip_j^{(2)}$ $(j=1, \cdots, m-k)$ are distinct respectively and infinitely differentiable with respect to (t, x, ξ') ($\lambda_i^{(1)}$ and $\lambda_j^{(2)}$ may coincide at some point for some i and j). Furthermore we assume that $\lambda_i^{(1)}(t, x, \xi)=\lambda_i^{(1)}(t, x, \xi|\xi|^{-1})|\xi|$ $(i=1, \cdots, k)$ satisfy the condition of M. Matsumura [8], that is