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ON THE ABSTRACT EVOLUTION EQUATION

Dedicated to Professor K. Shoda on his 60th birthday

By

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§ 0. **Introduction.** The present paper is concerned with the abstract evolution equation

$$du/dt+A(t)u = f(t)\,, \qquad 0\leq t\leq T\,, \tag{0.1}$$

in a Banach space X. $u=u(t)$ and $f(t)$ are functions on $[0, T]$ to X and $A(t)$ is a function on $[0, T]$ to the set of unbounded operators acting in X.

We have already published a number of papers on the integration of this equation based on the theory of semi-groups of operators; in particular the reader is referred to Kato [3] for a survey of recent results, including those obtained by other authors. In most (but not all) of these papers of ours, $-A(t)$ are assumed to be infinitesimal generators of *analytic semi-groups* $\exp(-sA(t))$ of bounded linear operators on X; this is equivalent to assuming that the resolvent $(\lambda I+A(t))^{-1}$ of $-A(t)$ covers a closed sector of the form $|\arg\lambda|\leq\frac{\pi}{2}+\theta$, $\theta>0$, and satisfies the inequality

$$\|(\lambda I+A(t))^{-1}\|\leq M/|\lambda|\,. \tag{0.2}$$

Regarding the dependence of $A(t)$ on t, it has so far been necessary to assume that the domain $D(A(t))$ of $A(t)$ or at least the domain $D(A(t)^h)$ of a certain fractional power $A(t)^h$ of $A(t)$ is independent of t, with other anxiliary assumptions such as the Hölder continuity of $A(t)^hA(0)^{-h}$ (see [3]).

The main object of the present article is to eliminate such an assumption on the domain of $A(t)$ or of $A(t)^h$. We shall prove the existence and the uniqueness of the solution of (0.1) or what comes essentially to the same thing, of the *evolution operator* $U(t, s)$ associated with (0.1); in addition to the condition that $-A(t)$ be the infinitesimal generator of an analytic semi-group, our principal assumption will be that an inequality