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On the Unknotted Sphere S^2 in E^4

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The construction of a locally flat, knotted sphere introduced by Artin [1] has given rise to a series of further investigations in this direction, [2], [3]. The construction is simply thus: Let E^2 be a plane in E^3 which is in turn in E^4, and let κ be a knot in E^3 having a segment ab in common with E^2, otherwise contained wholly in the positive half E^3_+ of E^3. Call the arc $\kappa^0 = \overline{\kappa - ab}$ an *open knot* with end points a, b. Artin obtained the desired sphere S^2 by rotating the open knot κ^0 around E^2 as axis in E^4. He showed that the fundamental group of $E^4 - S^2$ is isomorphic to the knot group of κ, that is, to the fundamental group of $E^3 - \kappa$. Fox and Milnor [4] showed that if a locally flat sphere S^2 in E^4

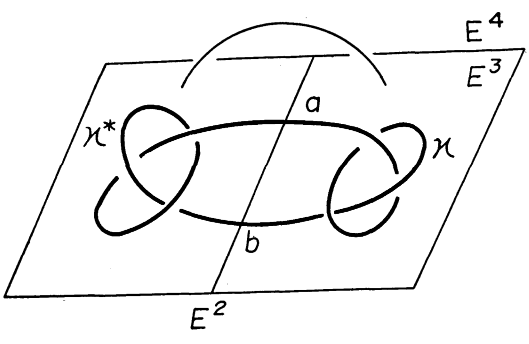

Fig. 1

is cut by an E^3, and if the intersection $S^2 \frown E^3$ is a knot, which they called a null-equivalent knot, then the Alexander polynomial of this knot must be of the form $f(x)f(x^{-1})x^n$. As it happens, the Alexander polynomial of $S^2 \frown E^3$ is $\Delta^2(x)$ for the sphere S^2 of Artin type, for then the knot in question is the product[1)] of κ, of Alexander polynomial $\Delta(x)$, with its symmetric image κ^* with respect to E^2, as will be seen in the figure.

Now the question is: what can be concluded about the knottedness of a given locally flat sphere $S^2 \subset E^4$ from the information about that of $S^2 \frown E^3$ for any hyperplane E^3 of E^4? This and other related questions

1) "sum" would be a better terminology.