

***Von Wright-Anderson's Decision Procedures  
for Lewis's Systems S2 and S3***

By Masao OHNISHI

In [1] A. R. Anderson described decision procedures for Lewis's system  $S4$  and for von Wright's system  $M$ . In this note similar procedures for Lewis's systems  $S2$  and  $S3^{1)2)}$  will be developed.

By virtue of the application of the results of my previous paper [4] not only the proof which shows the adequacy of our decision procedures will be considerably simplified comparing with [1], but also intrinsic interrelations between Gentzen's and von Wright-Anderson's methods will be made clear.

Familiarity with [1] and [4] will be presupposed.

**§1. Preliminaries.**

1.1. DEFINITION of *constituent* of a (modal) formula  $\alpha$  is as follows:

- (1) A propositional variable contained in  $\alpha$  is a constituent of  $\alpha$ .
- (2) A subformula (of  $\alpha$ ) of the form  $\Box\beta$  is a constituent of  $\alpha$ .

1.2. Construction of a truth-table for  $\alpha$ , denoted by  $\mathfrak{X}(\alpha)$ , the notion of *T-rows* and of *F-rows* of it, and the *value* of a subformula of  $\alpha$  in *Row* ( $i$ ) etc. are just the same as in [1].

**§2. A decision procedure for S2.**

2.1. DEFINITION. The number of the logical symbols  $\Box$  contained in a formula  $\alpha$  is called the *order* of  $\alpha$ .

2.2. DEFINITION.  $\alpha$  is an *E2-tautology* if and only if every *F-row* of  $\mathfrak{X}(\alpha)$ , denoted by *Row* ( $i$ ), satisfies at least one of the following two conditions:

I. Some constituent of the form  $\Box\beta$  has the value *T* in *Row* ( $i$ ), where  $\beta$  has the (assigned or calculated) value *F* in *Row* ( $i$ ).

II. Some constituents of the form  $\Box\gamma_1, \Box\gamma_2, \dots, \Box\gamma_n$  ( $n \geq 1$ ), all have

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1) Lewis and Langford [3].

2) Anderson reported in [1] (without detail) that he also solved the decision problem of  $S3$  in a similar way as [2]. But checking his unpublished solution the author is of opinion that it is incorrect.