On the Equations of Evolution in a Banach Space

By Hiroki Tanabe

§ 0. Introduction. In this paper, we again consider the equations of evolution

\[ \frac{dx(t)}{dt} = A(t)x(t) + f(t) \]  

and its associated homogeneous equation

\[ \frac{dx(t)}{dt} = A(t)x(t) \]

such as was treated in the previous papers [3] and [4]. However, we shall show that we can replace the strong continuous differentiability of \( A(t)A(s)^{-1} \) by its Hölder continuity by means of a slight change of the proof. It is quite clear that the differentiability of \( A(t)A(s)^{-1} \) is not necessary for the construction of the formal fundamental solution \( U(t, s) \) of (0.1'). In the previous papers, however, we used the differentiability essentially in appearance when we proved that the formal fundamental solution was really the desired one. So it is in this part that the modification of the proof is required. The inhomogeneous equation (0.1) can be treated similarly. Next, we shall give a generalization of a theorem of Solomijak concerning a perturbed equation

\[ \frac{dx(t)}{dt} = A(t)x(t) + B(t)x(t) + f(t) \]

Evidently, it is absurd now to consider (0.2) under the same assumptions about \( B(t) \) as in [3] and [4].

For the existence of the second derivative of \( U(t, s) \), it is sufficient to assume that \( A(t)A(s)^{-1} \) has a Hölder continuous derivative in \( t \).

§ 1. The fundamental solution of (0.1'). We denote by \( \Sigma \) a fixed closed sector which consists of those complex numbers \( \lambda \) satisfying

\[ -\theta \leq \arg \lambda \leq \theta, \quad \theta > \frac{\pi}{2} \]

plus the origin. Throughout this paper, we assume

Assumptions. For each \( t \) with \(-\infty < a \leq t \leq b < \infty\), \( A(t) \) is a closed operator with its domain dense in a Banach space \( \mathfrak{X} \) and its range

Tanabe, Hiroki
Osaka Math. J.
12 (1960), 363-376.