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On the Homology Group of Branched Cyclic Covering Spaces of Links

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Let k be a knot in 3-sphere S^3 and let $\mathfrak{M}_g(k)$ be the g-fold cyclic covering space of S^3, branched along k. By the use of the Alexander polynomial $\Delta(t)$ the 1-dimensional Betti number of $\mathfrak{M}_g(k)$ was calculated by L. Goeritz [2] and the product of the 1-dimensional torsion numbers has been calculated by R. H. Fox [1].

Now let $\mathfrak{L}$ be a link in 3-sphere. Then we can naturally define the g-fold cyclic covering space of S^3, branched along $\mathfrak{L}$ (see Section 2). The purpose of this paper is to calculate the product of the 1-dimensional torsion numbers and the 1-dimensional Betti number of this space. These will be done by the use of the ∇-polynomial defined by one of the authors of this paper [3] and the results are similar to the cases of knots (see Theorems 1, 2, and 3). The proof will be done similarly to [4] in the case of the product of torsion numbers and to [2] in the case of the Betti number.

Professor R. H. Fox kindly pointed out to us that the case of the product of torsion numbers is already proved in the thesis of J. P. Mayberry [5]. As J. P. Mayberry did not use the ∇-polynomial, his result is apparently different to that of ours. But these are essentially equivalent.

The calculation of the fundamental group of the complementary domain of the link represented in Sections 3 and 4 are due to G. Torres [8]. It is contained in this paper only for convenience of readers.

1. In this section we shall prove a lemma with respect to the determinant.

Let the $n\times n$ matrix X be

$$\begin{Vmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 1 \\ 0 & & & \ddots \\ T & 0 & \cdots & 0 \end{Vmatrix}$$

and the $n\times n$ matrix E be the unit matrix. By the simple calculation we have