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Some Properties of Complex Analytic Vector Bundles over Compact Complex Homogeneous Spaces

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Introduction

Recently R. Bott [7] has introduced the concept of homogeneous vector bundles over C-manifolds. By a C-manifold we understand here a simply connected homogeneous compact complex manifold, which was the subject of Wang's exhaustive work [22]. Bott's theory is a natural extension of the preceding researches by A. Borel and A. Weil [6] and permits us to utilize the theory of Lie groups to study complex analytic vector bundles over C-manifolds. The purpose of this paper is to study several problems on complex analytic vector bundles over C-manifolds utilizing Bott's results. The present paper is divided into four chapters. In Chapter I are given preliminaries for the subsequent chapters. First we recall the method of Y. Matsushima and A. Morimoto (cf. [16], [17]) to define the homogeneous (not necessarily vector) bundles in a more natural and intrinsic way. Their definition of homogeneous bundles does not presuppose the simply-connectedness of the base spaces, and in the case of C-manifolds it agrees with Bott's definition. Next, after a résumé of the results of Wang and Bott, we shall prove that every complex line bundle over a C-manifold is homogeneous (Theorem 1). This result was proved very recently by S. Murakami [18], but our proof is more direct and endows us some other implications. In Chapter II we discuss an application of the so-called classification theorem of complex analytic vector bundles to homogeneous vector bundles using Bott's idea (Theorem 2 and Theorem 4). The classification theorem of general vector bundles is due to S. Nakano, K. Kodaira and J. P. Serre (cf. [3], [19]), but it has not been yet published in a complete from and so we shall state our classification theorem of homogeneous vector bundles considerably in detail. In this chapter we shall show that the main parts of the researches by Borel-Weil [6] and by M. Gotô [9] are two special cases of our Theorem 2, and, as another application of this theorem, we shall show that the classical theorem of F. Severi concerning the positive divisors of complex Grassman manifolds (cf. [13]) can be generalized to any kählerian C-manifold of which the 2nd Betti number is 1 (Theorem 3). Chapter