

Decision Procedure for Modal Sentential Calculus S3

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Some trials to solve the decision problem for modal sentential calculus $S3^{1)}$ have been tried by W. T. Parry, S. Halldén, A. R. Anderson and some others. That is, in 1932, W. T. Parry [8] showed that γ^* is provable in $S3$ if and only if γ^* is provable in $S5$ where γ^* is of degree at most $1^{2),3)}$.

In 1950, S. Halldén [4] showed that the decision problem for $S3$ can be reduced to that for a new system $S7^{4)}$, which enlarges $S3$ by adjoining $\diamond\diamond p$ as an axiom to $S3$.

It is reported⁵⁾ that A. R. Anderson [1] solved the decision problem for $S3$ in 1953 using the method of von Wright [10].

The object of this paper is to give a Gentzen type decision procedure for modal sentential calculus $S3$.

The author wishes to express his cordial thanks to Mr. Masao Ohnishi for his suggestions and instructions in connection with this paper.

§1. Definitions of $Q3$ and Q^* and their equivalence.

Our formulations of $Q3$ and Q^* are based upon "Sequenzenkalkül LK ", which was constructed by G. Gentzen [3]. Namely:

- { logical symbols :
 - (and), ~ (not), \vee (or), \supset (if ... , then)
- { rules of inference (LK -rules) :
 - { structural rules
 - weakening, contraction, exchange and cut.
 - { logical rules
 - $(\rightarrow \cdot)$, $(\rightarrow \vee)$, $(\rightarrow \sim)$, $(\rightarrow \supset)$,
 - $(\cdot \rightarrow)$, $(\vee \rightarrow)$, $(\sim \rightarrow)$, $(\supset \rightarrow)$.

Numbers in brackets refer to the bibliography at the end of this paper.

- 1) C. I. Lewis and C. H. Langford [5].
- 2) For the definition of "a formula of degree n ", see A. R. Anderson [2], p. 203.
- 3) S. Halldén [4] remarked that γ^* is provable in $S2$ if and only if γ^* is provable in $S5$ where γ^* is of degree at most 1. See M. Ohnishi and K. Matsumoto [7], p. 119.
- 4) The decision problem for $S7$ has not been solved.
- 5) Recently Prof. Anderson wrote me the essential part of his solution for the decision problem of $S3$, but it seems to me that his solution is incorrect. (Added in proof.)