Terasaka, Hidetaka
Osaka Math. J.
12 (1960), 113–144.

On the Non-Triviality of Some Kinds of Knots

By Hidetaka TERASAKA

In our earlier paper [6] we have introduced a union of knots as an extension of the product of knots and proved among other things that a union of non-trivial knots is always non-trivial. A new class of knots with Alexander polynomial unity which Kinoshita introduced in the same paper, was proved there to be non-trivial by using the theorem of Bankwitz [3], [7], [4] that an irreducible alternating knot is non-trivial.

Now these two theorems referred to have to all appearance geometric character and purely geometric proofs have been desirable. The main purpose of the present paper is the presentation of geometric proofs of the above theorem with some extensions. In §1 is given a geometric proof of the non-triviality of torus knots, as well as that of parallel knots, in §2 that of doubled knots of Whitehead, which we have presented here not for the sake of their novelty but for completeness, because they were the starting point of the subsequent arguments. In the same § is given a rather lengthy proof of the theorem above mentioned that a union of non-trivial knots is non-trivial in somewhat extended form, and in §3 is proved a theorem giving sufficient conditions to determine from the projections of knots alone whether these are non-trivial. The rest of the paper is devoted to some applications of this theorem and among others a new kind of knots, called semi-alternating knots, is introduced as a generalization of irreducible alternating knots and proved that semi-alternating knots are always non-trivial (Theorem 6).

§1. Torus Knots and Parallel Knots.

The following considerations are based upon the semi-linear point of view.

1. Let κ_0 be a given knot, let $\dot{T}$ be a torus, i.e. a closed polyhedral surface of genus 1, with κ_0 as its core, and let T be the full torus bounded by $\dot{T}$. Span the knot κ_0 with a polyhedral surface F such that the intersection $F\frown T$ of F with the full torus T forms a ring bounded by κ_0 and a *closed line*, i.e. a simple closed polygonal line, $\xi_0=F\frown\dot{T}$ on the torus. ξ_0 is then a *basic longitude* of $\dot{T}$. Let η_0 be another closed