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A Singular Non-Linear Equation

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1. Introduction

To begin with, we wish to illustrate the physical problem which leads to the following mathematical work.

Let R be a region of three dimensional space occupied by an electrical conductor. Then each point in R becomes a source of heat as a current is passed through R. Let $u(x, t)$ be the temperature at the point $x \in R$ and at time t, and suppose that a function $E(x, t)$ which describes the local voltage drop in R is given as a function of position and time. Then if $\sigma(u)$ is the electrical resistivity which is, in general, a function of the temperature u, the rate of generation of heat at any point x at time t is $E^2(x, t)/\sigma(u)$. Let c and κ be the specific heat and thermal conductivity of R, respectively, which we take to be constant. Then the temperature satisfies the equation,

$$cu_t - \kappa\Delta u = E^2(x, t)/\sigma(u) ,$$

in the simplest case $\sigma(u) = \alpha u$ where α is a positive constant. More generally σ can be assumed to be a positive function of u which is increasing with u and which tends to zero with u. Thus the differential equation is singular in the sense that the right hand side becomes unbounded at $u=0$.

This physical problem leads naturally then to the study of the differential equation

$$u_t - \Delta u = F(x, t, u)$$

where Δ is the Laplace operator in E^N. We will write $Hu = u_t - \Delta u$ and call H the heat operator. Our equation then becomes

$$Hu = F(x, t, u) . \tag{1}$$

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