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On the Conditions of a Stein Variety

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§1. **Introduction.** The purpose of this paper is to give a criterion for a Stein variety. An analytic space $\mathfrak{V}$ [1] with a countable base is called a *Stein variety*, when:

1. $\mathfrak{V}$ is *holomorph-convex*; that is, a holomorphic convex hull of any compact subset of $\mathfrak{V}$ is compact. The holomorphic convex hull of a subset K is the set of the points P satisfying $|f(P)| \leq \text{Max}|f(K)|$ for all functions holomorphic in $\mathfrak{V}$.

2. For any two points $P, Q \in \mathfrak{V} (P \neq Q)$, there exists a function f holomorphic in $\mathfrak{V}$, such that $f(P) \neq g(Q)$.

3. For any point $P \in \mathfrak{V}$, there exists a finite number of functions holomorphic in $\mathfrak{V}$ which imbed a neighborhood U of P in the following way, i.e., by means of which U is represented as an analytic set[1] S in an open set of the space of complex variables of sufficiently high dimensions such that S has the property that, for arbitrary point P' of S, any function holomorphic in a neighborhood of P' is expressed as a trace of a function of the space[2].

The definition in this form is due to H. Grauert [2].

The problem of simplifying these conditions is treated by H. Grauert [2] and R. Remmert [7]. Grauert proved that a holomorphic convex analytic space (without the assumption of having a countable base) is a Stein variety, if it is K-complete. An analytic space $\mathfrak{R}$ is called *K-complete*, if, for any point $P \in \mathfrak{R}$, there exist a finite number of functions holomorphic in $\mathfrak{R}$ which map a neighborhood of P non degeneratedly at P, i.e., the image of P in the space of complex variables has as an inverse image in U a discrete set. Since, as Remmert remarked, K-completeness follows immediately from the separability condition, so, according to Grauert's result, one of the conditions (2., 3.) implies that a holomorph-convex analytic space is a Stein variety. But a holomorph-

1) Namely the set which is locally the common zeros of a finite number of equations.

2) In this paper, we shall call for convenience the conditions 2. and 3. the separability condition and the coordinate condition respectively.