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Stonian Spaces and the Second Conjugate Spaces of AM Spaces

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Let X be a compact space and let $C(X)$ be the set of all real-valued continuous functions on X. If any non-void subset of $C(X)$ with an upper bound has a least upper bound in $C(X)$, such a compact space X is called a stonian space.[1] Stone [10] has shown that a compact space X is stonian if and only if it is extremally disconnected, that is to say, if for any open set U in X its closure $\bar{U}$ is open. While, Kelley [9] has proved that if for any Banach space F containing a Banach space E there exists a projection of F on E whose norm is 1, E is isometric to $C(X)$, where X is stonian. Also Dixmier [4] has considered a compact space X such that $C(X)$ is isomorphic to an $L^\infty(R, \mu)$ as Banach algebras, where R is a locally compact space and μ is a positive measure on R. He called such a space X a hyperstonian space. A hyperstonian space is stonian. We shall see that a compact space X is hyperstonian if and only if $C(X)$ is lattice-isomorphic and isometric to a conjugate space of an AL space.[2]

In §1 we state some general properties of stonian spaces, and in §2 we consider an AM space $C(X)$ which is the second conjugate space of an AM space. Such a space X is hyperstonian, and if the character of X is countable, then X is the space βN_0, where N_0 is a discrete space whose cardinal number is at most countable (cf. Theorem 3, Corollary).

§ 1. Stonian spaces

For a completely regular space X, let βX denote the Čech compactification of X. (cf. Čech [2]). Dixmier [4] has shown that $\beta U = X$ for any open dense set U in a stonian space X. Therefore we obtain easily the following:

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1) See Stone [10] and Dixmier [4]. Numbers in bracket refer to the reference cited at the end of the paper.

2) See Kakutani [7] and [8].