

Gentzen Method in Modal Calculi

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A decision procedure by Gentzen style has been given by H. B. Curry [4] only for $S4$. J. Ridder [16] intended to give the decision procedures for M , $S4$ and $S5$ by Gentzen style¹⁾. By using the different methods, the decision procedures for the various modal systems have been obtained by J. C. C. McKinsey [12], R. Carnap [3], G. H. von Wright [17], J. Ridder [15], Alan Ross Anderson [1], and M. Itoh [8].

The object of this paper is to give decision procedures by Gentzen style for modal sentential calculi $S2$, $S4$, $S5$ and $M^2)$.

(I) Formulation

§1 Definitions of $S5^*$, $S4^*$, M^* and $Q2$.

1.1 Our formulation of the above systems is based upon "Sequenzkalkül LK ", which was constructed by G. Gentzen [6]. Namely :

$$\left\{ \begin{array}{l} \text{logical symbols :} \\ \quad \cdot \text{ (and), } \sim \text{ (not), } \vee \text{ (or), } \supset \text{ (if... , then)} \\ \text{rules of inference :} \\ \quad \left\{ \begin{array}{l} \text{structural rules} \\ \quad \text{weakening, contraction, exchange and cut.} \\ \text{logical rules} \\ \quad (\rightarrow \cdot) \text{ UES, } (\rightarrow \vee) \text{ OES, } (\rightarrow \sim) \text{ NES, } (\rightarrow \supset) \text{ FES,} \\ \quad (\cdot \rightarrow) \text{ UEA, } (\vee \rightarrow) \text{ OEA, } (\sim \rightarrow) \text{ NEA, } (\supset \rightarrow) \text{ FEA.} \end{array} \right. \end{array} \right.$$

Next, we add to LK two kinds of logical symbols :

$$\begin{array}{l} \diamond \dots \dots \text{(possible),} \\ \square \dots \dots \text{(necessary),} \end{array}$$

and we define as follows : if α is a formula, then $\diamond\alpha$ and $\square\alpha$ are also formulae.

Numbers in brackets refer to the bibliography at the end of this paper.

1) The authors have communicated to Prof. J. Ridder and he has admitted that his system was found to be unsatisfactory for the decision problem.

2) C. I. Lewis and C. H. Langford [9]. G. H. von Wright [17].