The Theory of Construction of Finite Semigroups I

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By a semigroup we mean a non-void set of elements x, y, z, $\cdots$ closed under an associative binary operation: $(xy)z = x(yz)$. If the number of elements of a semigroup S is finite, we call S a finite semigroup. The structure of a finite semigroup or a semigroup satisfying some conditions like finiteness was studied by Suschkewitsch, Clifford, Rees, and Schwarz [1], [2], [3], [4], [5]. However the classification of all finite semigroups and theory of construction of finite semigroups have not yet been discussed systematically. Though all types of semigroups of order 2, 3 and 4 were determined in [9], [10], the method was so unsystematic or inconvenient that it was not applicable to the general case. The author has investigated finite semigroups from the standpoint of a greatest semilattice decomposition, and this new method has been already applied to the case of order 5 [11].

Generally a semigroup is decomposed into a sum of special semigroups called s-indecomposable semigroups. Since s-decomposable semigroups are constructed out of a semilattice and s-indecomposable semigroups by means of the process of compositions, the study of finite semigroups is reduced to that of semilattices and s-indecomposable semigroups. Semilattices and s-indecomposable semigroups are subdivided into a few classes, and their construction must be considered in the different way from the case of s-decomposable semigroups.

In the series of these papers, we shall construct theoretically all finite s-decomposable semigroups and all finite s-indecomposable semigroups, except finite non-commutative simple groups, by induction with respect to the number of elements. Here, of course, the discussion on the construction of finite groups will be excluded, except only one case of c-indecomposable, non-simple groups.

The present paper is the first part of this series of the papers which consist of six parts:

    I   Greatest decomposition of a semigroup.

    II Compositions of semigroups and finite s-decomposable semigroups.

    III Finite c-indecomposable groups.