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On Regular Neighbourhoods of 2-Manifolds in 4-Euclidean Space. I

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Introduction

In 1921 L. Antoine [1] dealt with the embedding of sets in a Euclidean space R and he pointed out that there are three categories of the embeddings. Let P, Q be (topologically) equivalent sets in R. The first category: There is an orientation preserving homeomorphism onto $\psi : R \rightarrow R$ such that $\psi(P)=Q$. We say that P, Q are congruent. The second category: There are neighbourhoods $U(P)$, $U(Q)$ of P, Q respectively such that there exists an orientation preserving homeomorphism onto $\psi : U(P) \rightarrow U(Q)$ such that $\psi(P)=Q$. We say that P, Q are semicongruent. The third category: P, Q are neither congruent nor semicongruent.

The present paper deals with the second category of the piecewise linear embedding of polyhedral manifolds in R^n $(n=3, 4)$. The questions studied are mostly local in character. We often use some of the results and methods due to J.H.C. Whitehead [9] and V.K.A.M. Gugenheim [6. I, 6. II]. I am greatly indebted to their papers.

The exposition is as follows: In section 1 the results which are well known and will be used in the rest of the paper are stated. The results in section 2 are analogous appropriate to congruence of theorems due to Whitehead concerning regular neighbourhoods of polyhedra in a Euclidean space. Section 3 contains the fundamental definitions and lemmas. In section 4 we deal with $(n-1)$-manifolds in R^n $(n=3, 4)$ and show that the equivalent manifolds are semicongruent (Theorem 2). In section 5 we deal with 2-manifolds in R^4 (Theorem 3) and characterize the semicongruence classes of equivalent oriented 2-manifolds in R^4 (Theorem 4). Section 6 contains some geometric applications of the above considerations.

1. Preliminaries

1. 1. R^n will stand throughout this paper for n-dimensional metric

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