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On Algebras of Bounded Representation Type[1)]

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Let A be an associative algebra with a unit element over a field K and let $A=\sum_{\kappa=1}^{n}\sum_{i=1}^{f(\kappa)}Ae_{\kappa i}$ be a direct decomposition of A into directly indecomposable left ideals where $Ae_{\kappa i}\cong Ae_{\kappa 1}=Ae_{\kappa}$. It is well known that if A is generalised uniserial a directly indecomposable left module is homomorphic to one of $Ae_{\kappa}(\kappa=1, \cdots\cdots, n)$[2)]. But in general a directly indecomposable left (or right) module of A is not neceassarily homomorphic to Ae_{κ} and there may exist directly indecomposable left modules of arbitrary high degrees[3)]. In his paper [2] T. Nakayama propounded the problem to determine the general type of rings which possess arbitrary large directly indecomposable left (or right) modules and in [3] D. G. Higmann proved that every group has not indecomposable modular representations of arbitrary high degrees of characteristic p if and only if it has cyclic p-sylow subgroups.

Now in this paper we shall study necessary and sufficient conditions for an algebra to be of bounded representation type in a special case where $N^2=0$ (N is the radical of A)[4)] and K is algebraically closed.

First the *chain* $\{Ne_1, \cdots\cdots, Ne_s\}$ means that $Ne_{i+1}\neq Ne_i$ and Ne_{i+1} and Ne_i contain simple left ideals isomorphic to each other, namely $Ne_{i+1}\supset Au_{i+1}^{(\kappa)}$, $Ne_i\supset Au_{i+1}^{(\kappa)}$ and $Au_{i+1}^{(\kappa)}\cong Au_i^{(\kappa)}$. If Ne_1, Ne_2 and Ne_3 contain simple left ideals isomorphic to each other, namely $Ne_1\supset Au_1^{(\lambda)}$, $Ne_2\supset Au_2^{(\lambda)}$, $Ne_3\supset Au_3^{(\lambda)}$ and $Au_1^{(\lambda)}\cong Au_2^{(\lambda)}\cong Au_3^{(\lambda)}$, we define Ne_1 to be *divided* into Ne_2 and Ne_3. Then we shall prove that if $N^2=0$ and K is algebraically closed A has not directly indecomposable left (or right) modules of arbitrary high degrees if and only if the following conditions are satisfied;

(1) Ne_{λ} ($e_{\lambda}N$) ($\lambda=1, \cdots\cdots, n$) do not contain at least two simple

1) This means that the degree of the directly indecomposable representation is bounded. Cf. James P. Jans [4].

2) T. Nakayama [1].

3) H. Brummund [6].

4) If $N^2\neq 0$, it is very difficult and our conditions are extended as necessary conditions to the case where $N^2\neq 0$. But it is not proved yet that these conditions are sufficient for an algebra to be of bounded representation type. Cf. James P. Jans [4].