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On Compact Galois Groups of Division Rings

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On the subject of general non-commutative Galois theory, the present author has proved the existence of a fundamental correspondence between topologically closed regular subgroups and subrings for Galois division ring extensions with locally finite Galois groups,—in this case the groups are compact.[1] The main object of this paper is to give a necessary and sufficient condition for such compact Galois groups. In §1 it will be proved that a locally finite regular automorphism group is essentially outer, that is, can not contain but a finite number of inner automorphisms. Conversely we shall show in §2 that an essentially outer regular automorphism group is necessarily locally finite when the division ring extension is algebraic. At the same time, an extension theorem and a normality theorem will be proved for the Galois extensions in [7]. And lastly in §3 it will be proved that in the Galois extensions under the same assumptions any finite extensions are simply generated.

§1. Locally finite automorphism groups.

Let $\mathfrak{G}$ be a group of automorphisms of a division ring P. All the $\mathfrak{G}$-invariant elements of P form a subring Φ of P; in this situation we say that $\mathfrak{G}$ is an automorphism group of P/Φ.

DEFINITION. $\mathfrak{G}$ is said to be *locally finite* when each element of P is mapped by $\mathfrak{G}$ to at most a finite number of elements.

It is clear that, if there exists an automorphism group of P/Φ which is locally finite, then P is locally (left) finite over Φ, that is, any subring generated by Φ and a finite number of elements of P has a finite (left) rank over Φ. For, such a subring is always considered to be contained in a ring which has a finite automorphism group over Φ and the ring with a finite automorphism group over Φ has a finite rank over Φ.[2]

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1) See [7].

2) Its rank is not greater than the number of elements of the automorphism group. See [3].