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Indecomposable Completely Simple Semigroups Except Groups

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§1. If a semigroup S is homomorphic onto a semigroup T, a factor semigroup of S is obtained, namely, S is decomposed into a union of subsets by gathering elements of S mapped into the same element of T. Among all homomorphisms of S, there are two kinds of special cases: isomorphisms and a mapping of all elements of S to the one-element semigroup, which are called trivial homomorphisms. By an indecomposable semigroup we mean a semigroup without non-trivial homomorphism. As is well known a group is indecomposable if and only if it is simple. Of course finite semigroups of order at most 2 are indecomposable, and we shall call them as trivial cases. It is clear that an indecomposable semigroup has no proper ideal[1]. Otherwise we could consider Rees' difference semigroup of it modulo the proper ideal so that it would have a non-trivial homomorphism. In this paper we shall investigate a structure of indecomposable completely simple semigroups except groups [1].

According to Rees [1], a completely simple semigroup is represented as a regular matrix semigroup. In this paper, we shall use without special explanation the same terminology and notations as Rees'. Let G' denote a group G with zero 0 adjoined. Let P be an (M, L)-matrix, $(p_{\mu\lambda})$, $\mu \in M$, $\lambda \in L$, elements of which belong to G', satisfying the conditions that for any suffix $\mu \in M$ at least one $p_{\mu\lambda} \neq 0$, and that for any suffix $\lambda \in L$ at least one $p_{\mu\lambda} \neq 0$. Then a regular matrix semigroup S with a defining matrix P is defined to be a semigroup whose non-zero elements are all (L, M)-matrices $(x)_{\alpha\beta}$,[2] x varying over G, α over L, β over M, and the multiplication in S is defined as

$$(x)_{\alpha\beta}(y)_{\gamma\delta} = (xp_{\beta\gamma}y)_{\alpha\delta} .$$

In some cases S may contain a zero-matrix **0**, elements of which are all

1) By a proper ideal of S we mean a two-sided ideal distinct from S itself and from a set of only zero.

2) Denote by $(x)_{\alpha\beta}$ a matrix $X=(z_{\lambda\mu})$ where $z_{\lambda\mu}=x$ if $(\lambda, \mu)=(\alpha, \beta)$, and $z_{\lambda\mu}=0$ if $(\lambda, \mu)\neq(\alpha, \beta)$.