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On Homeomorphisms which are Regular Except for a Finite Number of Points

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Introduction

All spaces considered in this paper are separable metric. Let h be a homeomorphism of a set X onto itself. Then $p \in X$ is called *regular*[1] under h, if for each $\varepsilon > 0$ there exists $\delta > 0$ such that if $d(p, x) < \delta$, then $d(h^n(p), h^n(x)) < \varepsilon$ for every integer n. If $p \in X$ is not regular under h, then p is called *irregular*.

A set X will be called a *C^*-set* if $X - A$ is connected for any A which consists of a finite number of points of X. For example any n-manifold ($n \geqq 2$) is a C^*-set. Then one of the purpose of this paper is to prove the following

Theorem I. *Let X be a compact C^*-set and h a homeomorphism of X onto itself. If h is regular at every $x \in X$ except for a finite number of points, then the number of points which are irregular under h is at most two.*

We shall also prove the following

Theorem II.[2] *Let X be a compact C^*-set and h a homeomorphism of X onto itself such that*

(i) *h is irregular at a, b $(\neq) \in X$,*

(ii) *h is regular at every $x \in X - (a \cup b)$.*

Then either (1) *for each $x \in X - b$ $h^n(x)$ converges to a when $n \to \infty$ and for each $x \in X - a$ $h^n(x)$ converges to b when $n \to -\infty$, or* (2) *for each $x \in X - a$ $h^n(x)$ converges to b when $n \to \infty$ and for each $x \in X - b$ $h^n(x)$ converges to a when $n \to -\infty$.*

§ 1.

Let X be a set and h a homeomorphism of X onto itself. Let $R(h)$ be the set of all points which are regular under h and $I(h)$ the set of all points which are irregular under h. Then

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1) Introduced by B. v. Kerékjártó [5].

2) This is a converse theorem of Theorem 1 of the authors [3].