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On a Topological Characterization of the Dilatation in E^3

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Introduction

A topological characterization of the dilatation in E^2 has been given by B. v. Kerékjártó [5][1] and recently in another form by us [2]. The purpose of this paper is to give a topological characterization of the dilatation in E^3. In fact we shall prove the following

Theorem. *Let h be a homeomorphism of E^3 onto itself satisfying the following conditions:*

(i) *for each $x \in E^3$ the sequence $h^n(x)$ converges to the origin o when $n \to \infty$ and*

(ii) *for each $x \in E^3$ except for o the sequence $h^n(x)$ converges to the point at infinity when $n \to -\infty$.*

Then if h is sense preserving, h is topologically equivalent to the transformation

$$x' = \frac{1}{2}x,\ y' = \frac{1}{2}y,\ z' = \frac{1}{2}z$$

and if h is sense reversing, h is topologically equivalent to the transformation

$$x' = \frac{1}{2}x,\ y' = \frac{1}{2}y,\ z' = -\frac{1}{2}z$$

in Cartesian coordinates.

§1.

1. NOTATIONS. Throughout this paper h is a given homeomorphism of the 3-dimensional Euclidean space E^3 onto itself given by the assumption of our Theorem.

Following notations will be used:

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1) The numbers in the brackets refer to the references at the end of this paper.