Osaka Mathematical Journal
Vol. 6, No. 1, June, 1954

On the Existence of Unknotted Polygons on 2-Manifolds in E^3

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Introduction

All sets considered in this paper lie in the 3-dimensional Euclidean space E^3. Let P be a simple closed polygon and N an arbitrary set. Then P will be called an *N-unknotted polygon*, if P is the boundary-polygon of a polyhedral disk $D(P)$ whose interior is contained in N, and $D(P)$ will be called an *associated disk*. An E^3-unknotted polygon is usually called an *unknotted polygon*. The purpose of this paper is to prove the following theorems:

Theorem 1. *Let M be a closed polyhedral 2-manifold whose genus is different from* 0. *Then there exists an unknotted polygon on M not homologous to* 0 *in M.*

Theorem 2. *Let M be a closed polyhedral 2-manifold whose genus is differentfrom* 0. *Then there exists an $(E^3\text{-}M)$-unknotted polygon on M not homotopic to* 0 *in M.*

As an extension to Theorem 1 we have further

Theorem 3. *Let M be a closed polyhedral 2-manifold of genus p. Then there exists p mutually disjoint unknotted polygons such that they are linearly independent in the homology group of M.*

§ 1.

First we shall introduce several definitions.

Let M be a closed polyhedral 2-manifold. A family $\mathfrak{L}_\theta$ of the planes normal to a given unit vector θ will be said to be *admissible* with respect to M, if the vertices of M lie in different planes of the family. In general the intersection of M with representative plane L of an admissible family $\mathfrak{L}_\theta$ is the union of a finite number of mutually disjoint simple closed polygons. The intersection of M with an exceptional plane L of the family is either

(1) the union of an isolated point q and a finite collection of mutually disjoint simple closed polygons in the complement of q, or