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On Monomial Representations of Finite Groups

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In 1933 Shoda obtained remarkable results concerning monomial representations of finite groups [1]. Above all, he established a comprehensible criterion whether a transitive monomial representation of a finite group is irreducible or not, which is of general character; so that it is applicable to imprimitive representations of not necessarily finite groups. Further he proved the precise relation between the degree of a faithful irreducible representation of a metabelian group and the order of a maximal abelian normal subgroup containing the commutator subgroup. Giving alternative proofs to the above results of Shoda with some remarks, we shall show now the following

Theorem. *Every irreducible monomial representation of a finite group which is induced by its cyclic subgroup (which is different from the whole group) contains at least one not scalar diagonal matrix.*

§1.

First of all, for the completeness of the description, we give a proof to a theorem due to Frobenius [2]:

Proposition 1 (FROBENIUS). *Let G be an irreducible matrix group of finite order and let N be a normal subgroup of G. Let $N=r_1\Delta_1+\cdots+r_n\Delta_n$ be the irreducible decomposition of N. Then $r_1=\cdots=r_n$ and $\Delta_1,\cdots,\Delta_n$ are G-conjugate with each other.*

PROOF. We may assume, by the complete reducibility, that G is transformed into the form in which N is completely reduced: $N=\begin{pmatrix}\Delta^{(1)} & & \\ & \ddots & \\ & & \Delta^{(n)}\end{pmatrix}$, where $\Delta^{(1)}=r_1\Delta_1,\cdots,\Delta^{(n)}=r_n\Delta_n$. Let $X=\begin{pmatrix}X_{11} & \cdots & X_{1n}\\ \vdots & & \vdots \\ X_{n1} & \cdots & X_{nn}\end{pmatrix}$ be any matrix of G, where X_{ij} is of type $(\deg\Delta^{(i)}, \deg\Delta^{(j)})$ $(i,j=1,\cdots,n)$. Then we have

$$\begin{pmatrix}X_{11} & \cdots & X_{1n}\\ \vdots & & \vdots \\ X_{n1} & \cdots & X_{nn}\end{pmatrix}\begin{pmatrix}\Delta^{(1)}(Y) & & \\ & \ddots & \\ & & \Delta^{(n)}(Y)\end{pmatrix}=\begin{pmatrix}\Delta^{(1)}(XYX^{-1}) & & \\ & \ddots & \\ & & \Delta^{(1)}(XYX^{-1})\end{pmatrix}\begin{pmatrix}X_{11} & \cdots & X_{1n}\\ \vdots & & \vdots \\ X_{n1} & \cdots & X_{nn}\end{pmatrix}$$

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