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An Example of a Null-Boundary Riemann Surface

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We have proved that the Green's function is not[1] uniquely determined, when its pole is at an ideal boundary point of a null-boundary Riemann surface. M. Heins introduced[2] the notion of the minimal function due to R. S. Martin[3] and constructed a boundary point of dimension of preassigned number and conjectured that there would exist a boundary point of dimension infinity. We show by an example that his conjecture holds good.

1) Example. We denote by G the domain bounded by straight lines L_1, L_2 and the semi-circle C such that

$$L_1: 1 \leqq |z| < \infty, \quad \arg z = 0, \quad L_2: 1 \leqq |z| < \infty, \quad \arg z = \pi$$
$$C: |z| = 1, \quad 0 \leqq \arg z \leqq \pi.$$

On G we define a sequence of slits such that

$$I_1^i: 2^{i-1} \leqq |z| \leqq 2^i - \frac{1}{2^{i4}}, \quad \arg z = \frac{\pi}{2}: i = 2, 3, 4, \ldots$$

$$I_2^i: 2^{i-1} \leqq |z| \leqq 2^i - \frac{1}{2^{2i4}}, \quad \arg = \frac{\pi}{4}: i = 3, 4, 5, \ldots$$

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$$I_n^i: 2^{i-1} \leqq |z| \leqq 2^i - \frac{1}{2^{ni4}}, \quad \arg z = \frac{\pi}{2^n}: i = n+1, n+2, \ldots$$

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$$n = 1, 2, 3, \ldots$$

Let G^1 and G^2 be the same examplars with the same boundary and connect G^1 with G^2 by identifying L_1, L_2 and $\{I_j^i\}$ of them, to con-

1) Z. Kuramochi: Potential theory and its applications, I, Osaka Math. J. **3** (1951), 123–174.

2) M. Heins: Riemann surfaces of infinite genus, Annals of Math. **55** (1952), 296–317.

3) R. S. Martin: Minimal positive harmonic functions, Trans. Amer. Math. Soc. **19** (1941), 137–172.