Osaka Mathematical Journal
Vol. 5, No. 2, December, 1953

On Intuitionistic Functional Calculus

By Masao Ohnishi

This paper is separated into two parts; the first one is devoted to logical calculus, and the second one to the intuitionistic theory of real numbers based on Kuroda's treatment[1].

The next schema is due to A. Heyting[2]:

1. $\forall x F(x) \rightarrow \neg\neg \forall x F(x) \rightarrow \forall x \neg\neg F(x) \rightleftarrows \neg\neg \forall x \neg\neg F(x) \rightleftarrows \neg \exists x \neg F(x)$,
2. $\exists x F(x) \rightarrow \exists x \neg\neg F(x) \rightarrow \neg\neg \exists x F(x) \rightleftarrows \neg\neg \exists x \neg\neg F(x) \rightleftarrows \neg \forall x \neg F(x)$,
3. $\forall x \neg F(x) \rightleftarrows \neg\neg \forall x \neg F(x) \rightleftarrows \neg \exists x \neg\neg F(x) \rightleftarrows \neg \exists x F(x)$,
4. $\exists x \neg F(x) \rightarrow \neg\neg \exists x \neg F(x) \rightleftarrows \neg \forall x \neg\neg F(x) \rightarrow \neg \forall x F(x)$.

Explanations for the symbols used here: $\forall x$ and $\exists x$ are universal and existential quantifiers with respect to the individual variable x respectively. $F(*)$ is functional variable with certain (finite) number of arguments. $\rightarrow$ is one-way implication and $\rightleftarrows$ is (logical) equivalence of ante- and succedent formulae. (Hence these two are meta-logical symbols.) $\neg$ is negation and $\neg\neg$ is double negation of the remaining sub-formula after it.

1. Iterated quantifications.

Starting from the above schema by Heyting let us consider the case of iterated quantifications, where we shall be mainly concerned with the implicative relations between such formulae as follows: $\forall x\, F(x)$, $\exists x\, F(x)$, $\forall x\, \exists y\, F(xy)$, $\exists x\, \forall y\, F(xy)$, $\forall x\, \exists y\, \forall z\, F(xyz)$, $\exists x\, \forall y\, \exists z\, F(xyz)$ and their weakened forms to which $\neg\neg$'s are attached.

1. 1. Formulae with one quantifier. In this case the implicative relations are:

(1) $\forall x\, F(x) \rightarrow \neg\neg \forall x\, F(x) \rightarrow \forall x \neg\neg F(x) \rightleftarrows \neg\neg \forall x \neg\neg F(x)$,

(2) $\exists x\, F(x) \rightarrow \exists x \neg\neg F(x) \rightarrow \neg\neg \exists x\, F(x) \rightleftarrows \neg\neg \exists x \neg\neg F(x)$.

Hereafter some conventions will be used. The attached symbol $\neg\neg$

1) Kuroda (3).
2) Heyting (2).