Osaka Mathematical Journal
Vol. 5, No. 1, June, 1953

Some Remarks on the Uniform Space

By Kiyo AOTANI

In the theory of uniform space the main problems are: 1) the study of the compactness, 2) the study of the completion and 3) the study of Baire's property etc.. As for 2), Prof. K. Kunugui studied the uniform space replacing Weil's[1] condition by the local condition,[2] and concluded in this case that, if R is a uniform space and R^* is an imbedding of R in a complete space S, then the mapping from R to R^* is not only the bi-continuous but also uniformly bi-continuous mapping. But there remains in this case the problem to determine the complete space S depending only on Λ (Λ is the index set determining the uniform structure in R). In this paper we settle this problem and make some considerations about the local conditions.

§1. R is a neighbourhood space such that we can take, corresponding to each point p in R, a family $\{\mathfrak{D}_\lambda; \lambda \in \Lambda\}$ of fundamental systems of neighbourhoods. Then this space R is called a *uniform space* when $\{\mathfrak{D}_\lambda; \lambda \in \Lambda\}$ satisfies the following condition (α), where Λ is a set of indices and it is ordered by the condition that $\alpha \leqq \beta$ is possible if and only if for any element $V_\alpha(p)$ in $\mathfrak{D}_\alpha$ there exists some element $V_\beta(p)$ in $\mathfrak{D}_\beta$ and $V_\alpha(p) \subseteqq V_\beta(p)$ for each point p in R.

(α). Let p be any point in R; and if we select an arbitrary element $V_\lambda(p)$ from each $\mathfrak{D}_\lambda(\lambda \in \Lambda)$, then $\{V_\lambda(p); \lambda \in \Lambda\}$ is also a fundamental system of neighbourhoods of p.

(α) is nothing but a condition to agree with the topology of the neighbourhood space R,[3] and then the uniform space R with condition (α) satisfies Hausdorff's condition (B). Therefore the uniform space R is a T-space.

Then we assume that the uniform space R satisfies the following condition (W):

(W). For any index $\lambda \in \Lambda$ and any point s in R there exists some index $\mu = \mu(\lambda, s)$ in Λ such that if for three elements p, q and r in R we can take some neighbourhoods $V_1(r)$, $V_2(r)$ in $\mathfrak{D}_\mu$ that contain p and q respectively, then there exists always a neighbourhood $V_3(p) \in \mathfrak{D}_\lambda$ that contains q when s coincides with one of p, q and r.