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On Homotopy Type Problems of Special Kinds of Polyhedra II

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§ 1. Introduction

This paper is a continuation of my previous paper [14] of the same title, where I gave detailed accounts of homotopy types of *a* A_n^2-complex and of some special A_n^3-complex. They are completely determined by their cohomology groups, some homomorphisms μ, Δ, defined among them, and Steenrod's squaring operations, so that their homotopy invariants should be also determined by them. Homotopy type problems and related subjects are dealt with in this paper.

First, the exact sequence of J.H.C. Whitehead [4] is generalized in order to compute formally $\Gamma_{n+1}(0)$, $\Gamma_{n+2}(0)$ (§ 3) under some restrictions in dimensions. In case of cohomotopy groups this is accomplished by M. Nakaoka to get a generalization of the exact sequence of Spanier (refer to [15]). Utilizing this, we can compute up to group extension homotopy groups $\pi_{n+1}(P)$, $\pi_{n+2}(P)$ of a polyhedron P with vanishing homotopy groups $\pi_i(P)=0$ for each $i<n$. This calculation suggests us to compute combinatorially $\pi_{n+1}(P)$, $\pi_{n+2}(P)$ of an A_n^2-complex and also π_{n+2} of a special kind of polyhedron (see § 6). The study of reduced complexes in my previous paper and of J.H.C. Whitehead's secondary boundary operations (see § 4) enables us to solve thoroughly how $\pi_{n+1}(P)$※, $\pi_{n+2}(P)$ of an A_n^2-complex are computed by the aids of homology groups, of Steenrod's squaring homomorphisms, and of some homomorphisms μ, Δ, (see § 5), and also to get the way of calculation of $\Gamma_{n+1}(P)$, $\Gamma_{n+2}(P)$. In § 6 we restate concisely the results of my previous paper [14] through this sequence.

Until § 6 we assume $n>3$, or $n>2$.

We proceed to attack more complicated lower dimensional cases related to the subjects discussed till § 6. Recently Hirsch [16] gave a very elegant expression of the kernel of the homomorphism $i:\pi_3(P)\rightarrow H_3(P)$, where P is a simply connected polyhedron without 2-dimensional

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※ I have been informed of the existence of Hilton's paper on $\pi_{n+1}(P)$ through Chang's paper.