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# *On Homotopy Type Problems of Special Kinds of Polyhedra I*

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## 1. Introduction

It is one of the aims of modern topology to classify topological spaces by their homotopy types. Two spaces $X$ and $Y$ have the same homotopy type if there exist maps $f: X \rightarrow Y$ and $g: Y \rightarrow X$ such that $gf$ and $fg$ are homotopic to the identity maps $X \rightarrow X$ and $Y \rightarrow Y$ respectively. The problem of determining by means of invariants of $X$ and $Y$ whether $X$ and $Y$ are of the same homotopy type or not, is of great importance in modern topology. This general problem has not yet been solved. A number of particular results are well known.

In 1936 Witold Hurewicz solved in his famous paper [8]* the homotopy types of an $n$ dimensional locally connected compact metric space aspherical in dimensions less than $n$, and of a locally connected compact metric space aspherical in dimensions greater than unity. After this, many endeavours have been made to solve this general problem by several modern topologists, J. H. C. Whitehead, R. H. Fox, S. C. Chang, and others. Among them the recent brilliant results of J. H. C. Whitehead [3], [4] and of S. C. Chang [6] have much to do with the present paper. Whitehead reported in [3] that two simply connected, 4 dimensional polyhedra are of the same homotopy type if and only if their cohomology rings are properly isomorphic. According to Whitehead, an arcwise connected polyhedron $P$ is referred to as $A_n^2$-complex if dim. $P \leq n+2$ and $\pi_i(P)=0$ for each $i<n$. Though the author is unfortunate enough to be inaccessible to [4] here, he is informed of Whitehead's far reaching results through the introduction of Chang's paper [6]. They are stated as follows. Two $A_n^2$-complexes are of the same homotopy type if and only if their cohomology systems are properly isomorphic. Chang introduced new numerical invariants called secondary torsions to characterize the homotopy type of an $A_n^2$-polyhedron together with the Betti numbers and coefficients of torsion. Furthermore he reduced a given $A_n^2$-complex to a reduced complex which consists of elementary $A_n^2$-polyhedra.

The main purpose of this paper is to determine the homotopy type

* The number in square bracket is referred to the bibliography listed at the end of this paper.
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