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A Generalization of a Theorem of I. Kaplansky

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I. Kaplansky[1] showed that a bicompact space X is determined by the lattice of all continuous real functions on X. On the other hand it is proved by I. Gelfand, A. N. Kolmogoroff, M. H. Stone, E. Hewitt and others[2] that the rings of continuous real functions on spaces X, which are not always bicompact, determine spaces X.

In this note we discuss the characterization of some sublattices of lattices of all regular open sets on locally bicompact spaces and applying this characterization we show that locally bicompact spaces and complete metric spaces are determined respectively by sublattices of all continuous real functions on them. Furthermore we prove that e-complete spaces[3] and completely regular spaces are determined respectively by the lattices and the topological lattices of all continuous real functions on them.

§ 1. Lattices of regular open sets of locally bicompact spaces.

Definition 1. In a distributive lattice L with the smallest elements 0, we define a *binary relation* $a \supset b$ for two elements a and b of L as follows:

$$a \supset b \quad \text{if and only if} \quad a \wedge c \rightarrow b \wedge c = 0 .$$

Furthermore we say that *a is equivalent to b* if $a \supset b$ and $b \supset a$. Let $p(L)$ be the set of all equivalence classes and for two equivalence classes $[a]$ and $[b]$ we define $[a] \geqq [b]$ if $a \supset b$. Then $p(L)$ is also a distributive lattice.

Obviously $L = p(L)$ if and only if L satisfies Wallman's disjunction property[4] and $p(L) = p(p(L))$.

Definition 2. We say that a distributive lattice L with the smallest element 0 satisfying Wallman's disjunction property an *R-lattice*, if

1) I. Kaplansky [1].

2) I. Gelfand and A. N. Kolmogoroff [2], M. H. Stone [3], E. Hewitt [4], J. Nagata [5], D. Gale [6] and M. E. Shanks [7].

3) T. Shirota [11]. We say that completely regular space is e-complete if the structure with basis for uniformity made up of all countable normal coverings is complete.

4) H. Wallman [9].