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On a Two-Dimensional Space of Projective Connection Associated with a Surface in $\boldsymbol{R}_3$

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Denote by $\boldsymbol{R}_n$ an n-dimensional space of projective connection. First, in this paper, we treat the development of a curve in $\boldsymbol{R}_2$ by a method analogous to the theory on an ordinary projective plane curve. Next, we associate $\boldsymbol{R}_2$ with a surface S in $\boldsymbol{R}_3$ by a method of projection and investigate some properties of $\boldsymbol{R}_2$ and othei relations between $\boldsymbol{R}_2$ and $\boldsymbol{R}_3$.

1. Let $\boldsymbol{R}_n$ be an n-dimensional space of projective connection, in which a moving point is determined by a system of coordinates (u^i). If a natural frame[1] of reference $[A_0 A_1 \cdots A_n]$ is associated with the moving point A_0 in the tangential space of n dimensions at A_0 of $\boldsymbol{R}_n$, the infinitesimal displacement of the frame is given by

$$(1) \qquad dA_\alpha = \omega_\alpha^\beta A_\beta\,, \quad \omega_\alpha^\beta = \Pi_{\alpha k}^\beta\, du^k\,,$$

and

$$(2) \qquad \begin{cases} \omega_0^0 = 0\,, & \omega_0^i = du^i\,, \\ \Pi_{ik}^i = 0\,, & \Pi_{\beta 0}^\alpha = \Pi_{0\beta}^\alpha = \delta_\beta^\alpha\,, \end{cases}$$

where we denote by Greek letters α, β, etc. the indices which take the values $0, 1, \cdots, n$, and by Latin letters i, j, etc. those which take $1, 2, \cdots, n$.

Consider a curve C passing through A_0 of $\boldsymbol{R}_n$, where u^i are functions of a parameter t. Then we have along C

$$(3) \qquad \frac{dA_\alpha}{dt} = p_\alpha^\beta A_\beta\,, \quad \omega_\alpha^\beta = p_\alpha^\beta dt\,,$$

and

$$\frac{d^2 A_0}{dt^2} = p_0^i p_i^0 A_0 + \left(\frac{dp_0^i}{dt} + p_0^h p_h^i\right) A_i\,,$$

$$\frac{d^3 A_0}{dt^3} = \left\{\frac{d}{dt}\,(p_0^i p_i^0) + p_k^0\left(\frac{dp_0^k}{dt} + p_0^h p_h^k\right)\right\} A_0$$

$$+ \left\{\frac{d}{dt}\left(\frac{dp_0^i}{dt} + p_0^h p_h^i\right) + p_0^i p_0^h p_h^0 + p_k^i\left(\frac{dp_0^k}{dt} + p_0^h p_h^k\right)\right\} A_i\,,$$