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A Class of Topological Spaces

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1. Introduction. It is well known that the Čech's bicompactification $\beta(X)$ for any completely regular space X can be regarded as the completion of X in the uniform structure over X with the basis made up of all "finite" normal covering of X. In this point of view the following question naturally arises: What is the space which is obtained by the completion of the structure over X whose basis consists of all "countable" normal coverings of X?

In the present paper we are concerned with the space mentioned in the above problem. First of all we establish the relation between it and the Q-space introduced by E. Hewitt[1], then investigate the connections between our space and other important spaces. Moreover we discuss the relations between our space and the algebraic systems of the set of all continuous real valued functions on it.

2. Definition. Let us call the structure over a completely regular space X with the basis made up of all countable normal coverings of the space X the *e-structure* over X and denote by eX. Moreover we say the space with the complete e-structure to be *e-complete* and let us call a cardinal number $\mathfrak{m}$ *e-complete* if the discrete space with the potency $\mathfrak{m}$ is *e-copmlete*.

Remark. The notation "eX" was introduced by Tukey[2], but he said 'if the enumerable normal coverings are a basis for a uniformity, then we denote the uniformity by "eX"'. Thereby we shall show that the countable normal coverings are always a basis for a uniformity agreeing with the topology. To see that let X be a completely regular space and let $\mathfrak{U}$ be a countable normal coverings of X. Then we show that there exists a countable normal covering $\mathfrak{V}$ such that $\mathfrak{V} \overset{\Delta}{<} \mathfrak{U}$. Let $\mathfrak{U} = \{U_n\}$. Then since $\mathfrak{U}$ is normal, there exists an open covering $\mathfrak{U}_1$ such that $\mathfrak{U}_1 \overset{\Delta}{<} \mathfrak{U}$. For any i, let $F_i = X - S(X - U_i, \mathfrak{U}_1)$. Then $\{F_i\}$ is a closed covering of X such that $F_i \subset U_i$ for any i and such that

1) Cf. [5]
2) Cf. [8, p. 57]