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On Essential Components of the Set of Fixed Points

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Let X be a compact metric space and let f be a continuous mapping of X into itself. A fixed point p of f was called by M. K. Fort Jr.[1] an essential fixed point of f, if for every neighbourhood U of p there exists $\delta > 0$ such that every $g \in X^X$ with $|g-f| < \delta$ has at least one fixed point in U. Then for example, the identity mapping of the interval $[0, 1]$ has no essential fixed point. We shall introduce in this note a notion of essential components (see below) of the set of fixed points: thus if X is an absolute retract[2], then every continuous mapping of X into itself has essential components of the set of fixed points and if X is an absolute neighbourhood retract[3], then every continuous mapping of X into itself which is homotopic to a constant mapping has the same property.

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1. Let X be a compact metric space[4] and let f be a mapping[5] of X into itself. Let f have fixed points and let A be the set of all fixed points, C being a component of A. Then C will be called an *essential component* of A, if for every open set U which contains C there exists δ such that every $g \in X^X$ with $|g-f| < \delta$ has at least one fixed point in U. We say that X has *property* F' if every mapping of X into itself has at least one essential component of the set of fixed points.

Theorem 1. *Property F' is invariant under retraction*[6].

Proof. Let Y be a retract of a compact space X having property

1) M. K. Fort Jr.: Essential and nonessential fixed points, Amer. Jour. Math. 72 (1950), pp. 315-322.

2) In the sense of K. Borsuk. See, K. Borsuk: Sur let rétractes, Fund. Math. 17 (1931), pp. 152-170.

3) In the sense of K. Borsuk. See, K. Borsuk: Ueber eine Klasse von lokal zusammenhängenden Räumen, Fund. Math. 19 (1932), pp. 220-242.

4) In this note we assume that the space is separable metric.

5) In this note every mapping means a continuous mapping.

6) Let Y be a closed subset of X. If there exists a mapping r of X onto Y such that $r(x) = x$ for $x \in Y$, then Y is called by K. Borsuk a retract of X and the mapping r, a retraction of X onto Y. Cf. K. Borsuk, Fund. Math. 17. loc. cit.