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A Topological Characterization of Pseudo-Harmonic Functions

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Introduction. M. Morse and M. Heins[1] studied the relations among the zeros, poles and branch points of the "*pseudo-harmonic*" functions defined as follows:

Let $u(x, y)$ be a function which is harmonic and not identically constant in the neighbourhood N of a point (x_0, y_0) in $z(=x+iy)$-plane. Let the points of N be subjected to an arbitrary homeomorphism T in which N corresponds to another neighbourhood N' of (x_0, y_0) and the point (x, y) on N corresponds to a point (x', y') on N'.

Under T set

$$u(x, y) = U(x', y') \ .$$

Then the function $U(x', y')$ is called *pseudo-harmonic* on N'.

A function $U(x, y)$ is called *pseudo-harmonic* on a domain D, if $U(x, y)$ is pseudo-harmonic in some neighbourhood of each point of D.

We shall slightly extend the definition of the pseudo-harmonic function as follows:

Let F be a surface, i. e., a 2-dimensional and separable manifold. Let $U(p)$ be a real-valued function in the neighbourhood N of a point p on F, where N corresponds to $x^2+y^2<1$ in the z-plane by a homeomorphism $T(x, y)$.

Set

$$U(p) = U[T(x, y)] \equiv u(x, y).$$

Then $U(p)$ is called *pseudo-harmonic* in N, if $u(x, y)$ is harmonic and not identically constant. A function $U(p)$ is called *pseudo-harmonic* on F, if $U(p)$ is pseudo-harmonic in some neighbourhood of each point of F.

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1) M. Morse, The topology of pseudo-harmonic functions, Duke Math. Jour. 13 (1947) pp. 21-42. M. Morse and M. Heins, Topological methods in the theory of functions of a single complex variable, Annals of Math. 46 (1945), pp. 600-666, 47 (1946), pp. 233-274.