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Ergodic Skew Product Transformations on the Torus

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§ 1. Introduction

It is the purpose of this paper to give examples of ergodic transformations of some special types and to discuss their properties. We begin with the definition of *skew product measure preserving transformations*. Let φ be a measure preserving transformation on a measure space X. Let Y be another measure space, and let us assume that to every point x of the space X, there corresponds a measure preserving transformation ψ_x on Y. Let Ω be the direct product measure space of X and Y:

$$\Omega = X \times Y, \quad \omega = (x, y), \quad \omega \in \Omega, \quad x \in X, \quad y \in Y.$$

Denote the measures on X, Y and Ω by m, μ and ν respectively, ν is the completed direct product measure of m and μ.

If the family of measure preserving transformations $\{\psi_x | x \in X\}$ satisfies certain measurability conditions, it is easy to see that the transformation T which is defind by

$$T(x, y) = (\varphi x, \psi_x y)$$

is a measure preserving transformation on Ω. Then T is called a *skew product measure preserving transformation*. In case the family of transformations $\{\psi_x\}$ consists of the same transformation ψ, the skew product transformation T is the direct product transformation of φ and ψ.

In this paper we assume that φ is an *ergodic* measure preserving transformation on a measure space X, and that Y is the usual Lebesgue measure space of the set of real numbers mod 1, which will be called simply *circle*.

Let A be the set of all Y-valued measurable functions on X. To any $\alpha(x)$ belonging to A we may assign a one-to-one mapping T on Ω in the following way:

$$T(x, y) = (\varphi x, \alpha(x) + y).$$