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Note on Locally Compact Groups

By Hidehiko YAMABE

§ 1. The purpose of this note is to study the problem proposed by C. Chevalley: Is it true that a locally compact group which has no arbitrarily small[1] subgroup is a Lie group?

Concerning the above problem two theorems will be proved in this note. One of them is:

Theorem 1. *A locally euclidean group G which has a neighbourhood of the identity containing no non-trivial subgroup, has a neighbourhood $\tilde{U}$ of the identity, through any point of which there exists one and only one one-parameter subgroup*[2].

The other is:

Theorem 2. *If $(U_n)^n$ is contained in $\tilde{U}$, then G is a Lie group, where U_n denotes the aggregate of the n-th roots of elements in a neighbourhood U.*

§ 2. For an element x of a neighbourhood U of the identity e we denote by $\delta_U(x)$ the smallest number n such that $x^{2^n} \in U$. The group G is said to have the property (S) if there exists a neighbourhood U of e such that $\delta_U(x) < \infty$ for every x in $U - \{e\}$. According to Kuranishi[3] a locally euclidean group G which has the property (S), has a neighbourhood of the identity e, through any point of which one can draw one and only one one-parameter subgroup. Therefore we have only to show that a locally compact group which has no arbitrarily small subgroup has the property (S).

In order to prove this we shall need the following Lemma.

Lemma 1. Let W be a neighbourhood of the identity e which contains no non-trivial subgroup in it. For an arbitrarily small neigh-

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1) A small subgroup means a subgroup contained in a sufficiently small neighbourhood of the identity.

2) This theorem was proved with the co-operation of Dr. Gotô. Cf. the forthcoming Nagoya Math. Journal.

3) See Kuranishi: *Differentiability of locally compact groups*, Nagoya Math. Journal Vol. 1, 1950, 71-81.