Osaka Mathematical Journal
Vol. 3, No. 1, March, 1950

Remarks on the Postulates of Metric Groups

By Zen'ichiro KOSHIBA

§ 1. Introduction

Let E denote a topological space, which is an abstract group at the same time. But we do not mean by E a topological group in the ordinary sense. In this note we shall discuss about some relations among the following postulates under the condition of metric completeness, or under that of metric local compactness:

(1) *If* $\lim x_n = x$, *then* $\lim x_n y = xy$,

(2) *if* $\lim y_n = y$, *then* $\lim x y_n = xy$,

(3) *if* $\lim x_n = x$ *and* $\lim y_n = y$, *then* $\lim x_n y_n = xy$,

(4) *if* $\lim x_n = u$ *and* $\lim x_n^{-1} = v$, *then* $u^{-1} = v$,

(5) *if* $\lim x_n = x$, *then* $\lim x_n^{-1} = x^{-1}$.

Our results are the following two theorems:

Theorem I: *If E is a metric complete group, then the property* (3) *can be deduced from* (1) *and* (2).

Theorem II: *If E is a metric lacally compact group, then the property* (5) *can be deduced from* (1) *and* (2), *and E is a metric locally compact group in the ordinary sense.*

BANACH gave in his postumas note [1] a theorem that a metric complete group satisfying (1), (2) and (4) has the property (5). From it and Theorem I follows Theorem II (even in the case of metric completeness instead of metric locally compactness) as their logical consequence. But his proof in the non-separable case is not evident for us. In the following, let "e" denote the unit element of the group E, V_e or W_e spherical neighborhood of e, $S_r(x)$ the spherical neighborhood of $x (\in E)$ with radius r, and $d(x, y)$ the distance between x and y. $\bar{S}_r(x)$ means the closure of $S_r(x)$.

§ 2. Proof of Theorem I.

Before the proof of theorem I we shall prove the following:

---

1) Remarques sur les groupes et les corps métriques, Studia Math. 10 (1948), p. 178.