Osaka Mathematical Journal,
Vol. 2, No. 2, November, 1950

# *On the Independence of Quadratic Forms in a Non-Central Normal System* *

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The problem on the independence of quadratic forms in a normal system originates from the famous treatise of W. G. Cochran[1]. Cochran proved the following theorem: "Let $x_1, x_2, \dots, x_n$ be independently distributed according to the identical normal law $N(0,1)$[2], $q_1, q_2, \dots, q_m$ being $m$ quadratic forms of $x_1, x_2, \dots, x_n$, and their ranks $r_1, r_2, \dots, r_m$ respectively. If $\sum_1^m q_j = \sum_1^n x_i^2$, then the necessary and sufficient condition for $q_1, q_2, \dots, q_m$ to be independent statistically is that $\sum_1^m r_j = n$. When these conditions are satisfied, then $q_1, q_2, \dots, q_m$ are distributed independently according to the chi-square distributions of degrees of freedom $r_1, r_2, \dots, r_m$ respectively". Then, in 1940, W. G. Madow[3] proved the generalization of this theorem for the non-central case, and he obtained the same condition.

On the other hand, A. T. Craig[4], H. Hotelling[5] and H. Sakamoto[6] have extended the theorem in the other direction, their point being as follows: Let $x_1, \dots, x_n$ be independently distributed according to the normal law $N(0,1)$ and written as a vector $\mathfrak{x}=(x_1, \dots, x_n)$, and furthermore let $A$ and $B$ be two real ymmetric matrices, then the necessary and sufficient condition for two quadratic forms $q_1 = \mathfrak{x}A\mathfrak{x}'$ and $q_2 = \mathfrak{x}B\mathfrak{x}'$ to be independent statistically is $AB=0$. But their proofs were insufficient, and K. Matsushita[7] and we[8] gave the complete proofs.

In this note we shall generalize the last theorem for the non-central case, and prove the following two theorems and show one example of their applications.

## § 1. THEOREMS.

**Theorem I.** Let $x_1, x_2, \dots, x_n$ be normally and independently distributed with means $a_1, a_2, \dots, a_n$ respectively and with the common variance unity. If we denote $n$ random variables $x_1, x_2, \dots, x_n$ by a vector notation $\mathfrak{x}=(x_1, x_2, \dots, x_n)$, and make two quadratic forms $q_1=$