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Note on the Markoff's Theorem on Least Squares [1]

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The purpose of this note is to give a simple proof of the extension of the famous Markoff's theorem on least squares by J. Neyman [2] and F. N. David, which is very useful especially in the theory of sampling [3].

Theorem. Let n random variables $x_1, x_2, \dots, x_n$

(a) be independently distributed [4], and

(b) their means be linearly restricted with $s(\leq n)$ unknown parameters $p_1, p_2, \dots, p_s$ with known coefficients, i. e.

$$E(x_i)=a_{i1}p_1+a_{i2}p_2+\dots+a_{is}p_s,\ i=1, 2, \dots, n, \tag{1}$$

where the coefficients $a_{ij}, i=1, 2, \dots, n;\ j=1, 2, \dots, s$ are known constants.

(c) The rank of the coefficient matrix

$$A=\begin{pmatrix} a_{11}a_{12}\cdots\cdots a_{1s} \\ a_{21}a_{22}\cdots\cdots a_{2s} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ a_{n1}a_{n2}\cdots\cdots a_{ns} \end{pmatrix} \tag{2}$$

is equal to s.

(d) Further, let the variance σ_i^2 of x_i be

$$\sigma_i^2=\frac{\sigma^2}{P_i},\quad i=1, 2, \dots, n, \tag{3}$$

where $P_1, P_2, \dots, P_n$ are known constants and σ unknown.

If the above conditions are satisfied, then the follnwing two statements (α) and (β) hold.

(α) The best unbiased linear estimate [5] of the linear form

$$\theta=b_1p_1+b_2p_2+\dots+b_sp_s \tag{4}$$

with known coefficients $b_1, b_2, \dots, b_s$ is

$$F=b_1p_1^0+b_2p_2^0+\dots+b_sp_s^0, \tag{5}$$