

On Systems of Structures of a Completely Regular Space

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Considering the set of all structures¹⁾ over a completely regular space R , we propose to investigate the connections between the algebraic properties of the set and the topological properties of R . To this purpose we introduce in the following an order into the set in question and characterize among others the space R which has either the maximum or the minimum for certain subsystems of the ordered system thus obtained, and determine further the cardinal numbers of the ordered systems in the case when R is metrizable.

For the notations we follow those used by Tukey¹⁾.

§ 1. Definitions and remarks.

DEFINITION. Let gR be a structure over a completely regular space R with a basis $\{\mathfrak{U}_\alpha | A\}$ and let $g'R$ be another structure over the space R with a basis $\{\mathfrak{B}_\beta | B\}$, then we write $gR \geq g'R$, if for any $\mathfrak{B}_\beta \in \{\mathfrak{B}_\beta | B\}$ there exists $\mathfrak{U}_\alpha \in \{\mathfrak{U}_\alpha | A\}$ such that $\mathfrak{B}_\beta > \mathfrak{U}_\alpha$, i. e., if the identical mapping of R on itself is a uniformly continuous mapping of gR onto $g'R$.

It is clear that the set of all structures over a completely regular space R is considered as an ordered system in this order. Let us denote by $D(R)$ this ordered system and let $D_t(R)$ be the subsystem of $D(R)$ which consists of all totally bounded structures over R . Let m be an infinite cardinal number. We say that a structure gR is an m -structure, if the uniformity of gR contains a basis $\{\mathfrak{U}_\alpha | A\}$ with $|A| \leq m$ ²⁾, and we denote by $D_m(R)$ the set of all m -structures over R .

Evidently $D(R)$, $D_t(R)$ and $D_m(R)$ are not only ordered systems, but also directed systems and $D(R)$ and $D_t(R)$ have always the maximal element. Moreover an \aleph_0 -structure is a metric space.

1) John W. Tukey, Convergence and uniformity in topology (1940). We will use "structure" to mean "struct" used by Tukey and in this note we will refer this book by the letter T.

2) The cardinal number of a set A will be denoted by $|A|$ and small German letters will be used for infinite cardinal numbers.