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On a Local Property of Absolute Neighbourhood Retracts

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1. In this note we shall prove the following theorem and derive from it several corollaries.

THEOREM. *In order that a separable metric space Y is an absolute neighbourhood retract*[1]) *it is necessary and sufficient that Y is compact and has the following property (L):*

(L) *For each point $p \in Y$ and its neighbourhood U there exists a neighbourhood $V \subset U$ of p such that every continuous mapping f of a closed subset A of a given metric space X into V can be extended over X with respect to U.*

This theorem is an extension of the C. Kuratowski's characterization[2]) of absolute neighbourhood n-retracts and gives a local property of absolute neighbourhood retracts.

2. First of all let us prove[3]) the necessity of the condition. Suppose Y is an absolute neighbourhood retract. Y may be considered[4]) as a neighbourhood retract of Hilbert parallelotope Q. Let r be the retraction. Then $r^{-1}(U)$ is an open set of Q containing p. Let ε be a positive number and

$$K_\varepsilon = \underset{x \in Q}{E}[\rho(x, p) < \varepsilon].$$

Take $\varepsilon > 0$ sufficiently small such that $\bar{K}_\varepsilon \subset r^{-1}(U)$ and put $V = K_\varepsilon Y$. Each mapping $f \in V^A$ has an extension $f_1 \in Q^X$[5]). Let π be the projection of Q onto $\bar{K}_\varepsilon$ such that

1) In the sense of K. Borsuk. See, K. Borsuk: Über eine Klasse von lokal zusammenhängenden Räumen. Fund. Math. 19 (1932), pp. 220-242.

2) C. Kuratowski: Sur les espaces localement connexes et péaniens de dimension n. Fund. Math. 24 (1935), p. 273, Théorème 1.

3) The proof of C. Kuratowski also holds in the case $n=\infty$ without the assumption of compactness of Y. But if Y is compact we can prove it more simply as in the text.

4) K. Borsuk, loc. cit. p. 223, Section 3.

5) W. Hurewicz and H. Wallman: Dimension Theory, p. 82, Cor. 1.