

**Unitary representations of locally compact groups**  
— **Reproduction of Gelfand-Raikov's theorem** —

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1. INTRODUCTION. It is reported in Mathematical Reviews that I. Gelfand and D. Raikov proved, in 1943, that every locally compact group  $G$  admits sufficiently many irreducible continuous unitary representations on Hilbert spaces (that is, for any element  $g$  of  $G$  different from its identity element, there exists a representation of  $G$  which maps  $g$  to an operator different from the identity one) [3]<sup>1)</sup>. For the present author it is yet impossible to read their paper, but he had fortunately the opportunity to read the papers [6, 7] by I. E. Segal and was suggested the original direct proof by Gelfand and Raikov. The purpose of the present paper is to reproduce this direct proof.

Later the paper on the same subject by R. Godement [4] became available to the present author. The contents of the present paper are, in essential, involved in this paper.

2. SUMMARY. After some preliminary remarks in 3, we shall, in 4, establish the correspondence between measurable positive definite functions on  $G$  and so-called simple continuous unitary representations of  $G$  on Hilbert spaces. There we shall make use of the group algebra as [2], [6] and [7] do<sup>2)</sup>. Every continuous unitary representation of  $G$  is obtained by the method used in 4, if, analogously to the finite dimensional case, we regard two representations as equivalent when they are mutually transformed by a unitary operator. In 5, we shall consider the correspondence between irreducible representations and so-called extreme positive definite functions and prove Gelfand-Raikov's theorem

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1) Number in LITERATURE at the end of this paper.

2) We shall also utilize, as [7] does, some results in [1], which is reviewed in Math. Reviews but, to the present author, is yet unavailable.