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On hypergroups of group right cosets

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In this paper we present certain results for the so-called hypergroups of classes, or more precisely, hypergroups of (group) right cosets. In § 1. we give several definitions. For any hypergroup of right cosets we give in § 2. a representation by permutations which will be used to characterize such hypergroups. By means of some partitions of elements of a hypergroup of right cosets we may define new hypergroups of right cosets which are treated in § 3. Some results on such kind of partitions for cogroups are given in § 4. This investigation is applied to obtain a counter-example for the conjecture of J. E. Eaton that every cogroup is isomorphic to a hypergroup of right cosets. The author expresses many thanks to Prof. K. Shoda for his kind encouragement and valuable remarks.

§ 1.

A set M is called a *hypergroupoid* if a product ab is defined to be a non-empty subset of M for every a and b in M. We define the product ST for any two subsets S and T of a hypergroupoid M as the set-sum of all products st of s in S and t in T. An element e of M satisfying the relation $ae \ni a$ for any a in M is called a *right unit* of M. Similarly we define a *left unit* and a *two-sided unit*. A one-to-one mapping θ of M onto itself is called a *(right) multiplicator* of M if $ab \ni c$ implies $ab^{\theta} \ni c^{\theta}$ and conversely. The totality of multiplicators of M forms an ordinary group which will be denoted by $R(M)$. A subgroup T of $R(M)$ is called a *(right) transferor group* of M if it satisfies the condition: if $ab \ni b'$ and $ac \ni c'$ then there exists a mapping θ in T such that $b^{\theta} = c$ and $b'^{\theta} = c'^{\theta}$. If T is a transferor group of M then any group U between T and $R(M)$ is also a transferor group of M.

Let M and N be two hypergroupoids. A many-to-one mapping θ of