Ôsaka Mathematical Journal
Vol. 1, No. 1, March, 1949

Contribution to the problem of stability

By Tatsuji KUDO

I. INTRODUCTION

1. H. Hopf and E. Pannwitz[1]) introduced the notion of *stability*[2]) and raised several questions, the most important of which reads: "Can we characterize the stability in terms of the homology theory ?".

Confining themselves to the homogeneous n-complex K^n, they obtained the following theorems:

THEOREM *A*. A linear graph K^1 is stable if and only if it has no free side.

THEOREM *B*. A cyclic[3]) complex K^n is stable for any dimension n.

THEOREM *C*. For $n \geqq 3$ a stable complex K^n is cyclic, provided that it is simply connected.

Recently Professor A. Komatu has reasonably conjectured that to these theorems can be given the following complete and unified form:

THEOREM *D*. For a locally finite homogeneous n-complex K^n $(n \neq 2)$ stability is equivalent to the cyclicity in the sense of local coefficients[4]).

The main purpose of this paper is to prove THEOREM D by generalizing the Hopf-Pannwitz's lemmas based on ordinary coefficients to those based on local coefficients.

---

1) H. Hopf and E. Pannwitz, Über stetige Deformationen von Komplexen in sich, Math. Ann., 108, 1932.

2) A topological space R is called stable if for every deformation f_t of R through itself no point of R can get rid of the covering by the image $f_1(R)$, or more simply, if R can never be deformed into its proper subspace.

3) An n-complex K^n is called cyclic, if each n-simplex σ_i^n is contained in at least one n-cycle with suitable coefficients.

4) See, IV. § 8.