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ON MANIFOLDS WITH TRIVIAL LOGARITHMIC TANGENT BUNDLE

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1. Introduction

By a classical result of Wang [15] a connected compact complex manifold X has holomorphically trivial tangent bundle if and only if there is a connected complex Lie group G and a discrete subgroup Γ such that X is biholomorphic to the quotient manifold G/Γ. In particular X is homogeneous. If X is Kähler, G must be commutative and the quotient manifold G/Γ is a compact complex torus.

The purpose of this note is to generalize this result to the non-compact Kähler case. Evidently, for arbitrary non-compact complex manifold such a result can not hold. For instance, every domain over $\mathbb{C}^n$ has trivial tangent bundle, but many domains have no automorphisms.

So we consider the "*open case*" in the sense of Iitaka ([7]), i.e. we consider manifolds which can be compactified by adding a divisor.

Following a suggestion of the referee, instead of only considering Kähler manifolds we consider manifolds in class $\mathcal{C}$ as introduced in [5]. A compact complex manifold X is said to be class in $\mathcal{C}$ if there is a surjective holomorphic map from a compact Kähler manifold onto X. Equivalently, X is bimeromorphic to a Kähler manifold ([14]). For example, every Moishezon manifold is in class $\mathcal{C}$.

We obtain the following characterization:

Main Theorem. *Let $\bar{X}$ be a connected compact complex manifold, D a closed analytic subset and $X = \bar{X} \setminus D$. Assume that $\bar{X}$ is in class $\mathcal{C}$ as introduced in* [5] (*also called* "*weakly Kähler*").

Then the following conditions are equivalent:

(1) *D is a divisor which is locally s.n.c.* (*see definitions in* §2 *below*) *and the logarithmic tangent bundle $T(-\log D)$ is a holomorphically trivial vector bundle on $\bar{X}$.*

(2) *There is a complex semi-torus G acting effectively on $\bar{X}$ with X as open orbit such that the all the isotropy groups are themselves semi-tori.*

Moreover, if one (*hence both*) *of these conditions are fulfilled, then D is*

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